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# Angular Misalignments between Probe Sections of the 7m Integral Probe for LHC Quadrupole Field Measurement

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The relative angular offsets between the 3 shaft sections of the integral probe used for cold measurements of LHC quadrupoles have been calibrated (given below). I discuss the magnitude of the offsets relative to expectations and consider the effect on field quality measurements. I also take up the general question of twist and misalignment.

## *Probe angular alignment*

The 3 probe sections were calibrated in a short corrector dipole so that each section could be considered independently. The phase returned by each section is given in the follow table.

| section | phase (rad) | differences (wrt avg) |
|---|---|---|
| 1 | -0.585 | 0.0028 |
| 2 | -0.605 | -0.0173 |
| 3 | -0.573 | 0.0145 |
| **avg.** | -0.588 | |
| **st. dev.** | | 0.01611 |
| **sum of squares** | | 0.00052 |

## *Probe assembly and alignment*

The probe was actually constructed from 6 pieces of machined G10 tube. The 3 sections each consist of two pieces glued together. During the gluing process, alignment of the two was made by inserting shim stock into the corresponding slots of the pieces. Shims were inserted for the slots in the ~1/4 of the circumference to insure stability and integrity of the match during the glue up. Alignment of the 3 sections was made in the same manner. However, alignment was complicated by the presence of the bearing housings between the sections. The alignment between sections is certainly not as good as that between the two pieces in each section which should be good to a fraction of the slot (0.01 in./0.25mm). The angular misalignments (between sections 1 and 2 or 2 and 3) above correspond to arc lengths of 0.41 and 0.65 mm respectively – a displacement on the surface of the probe on the order of a couple of slot widths. Given that the alignment must be made over the bearing housing across a gap of 4 cm, this level of alignment is more or less as expected given the methods used.

## *Effect of misalignment on field quality measurements*

This analysis is similar to the treatment of a random twist of the probe body in [1]. In this case we have a discrete phase in the different probe sections rather than one that is continuously changing. We characterize the phase by the average phase of the (3) sections $\delta_c$ and the difference from the average in section *i* of $\varepsilon_i$. The flux though the tangential coils is given by

$$\Phi_n(t) \propto \frac{1}{L}\int_0^L C(n)\sin(n\omega t + n\delta_c + n\varepsilon(z) + n\alpha_n)dz.$$

Using the sum of angles formula for sin(A+B) and then the power series expansion of sin(nε) and cos(nε) keeping the terms of second order or less - ε(z) being small) - we get the following expression for the flux.

$$\Phi_n(t) \propto \frac{1}{L}\left[\int_0^L C(n)\sin(n\omega t + n\delta_c + n\alpha_n)(1-\frac{(n\varepsilon(z))^2}{2})dz + \int_0^L C(n)\cos(n\omega t + n\delta_c + n\alpha_n)n\varepsilon(z)dz\right].$$

The first integral has no z dependence and becomes

$$\Phi_n(t) \propto C(n)\sin(n\omega t + n\delta_c + n\alpha_n).$$

This is the usual angular expression in the sensitivity factor. The third term

$$\Phi_n(t) \propto \frac{1}{L}C(n)\cos(n\omega t + n\delta_c + n\alpha_n)n\int_0^L \varepsilon(z)dz = 0$$

as the integral of ε over the length of the probe is by necessity zero. Evaluating the second term, we get

$$\Phi_n(t) \propto -\frac{1}{L}C(n)\sin(n\omega t + n\delta_c + n\alpha_n)\frac{n^2}{2}\int_0^L \varepsilon(z)^2 dz$$

$$= -\frac{1}{L}C(n)\sin(n\omega t + n\delta_c + n\alpha_n)\frac{n^2}{2}\left[\int_0^{L/3}\varepsilon_1 dz + \int_{L/3}^{2L/3}\varepsilon_2 dz + \int_{2L/3}^{L}\varepsilon_3 dz\right]$$

$$= -\frac{1}{L}C(n)\sin(n\omega t + n\delta_c + n\alpha_n)\frac{n^2}{2}\left(\frac{L}{3}\sum_{i=1}^{3}\varepsilon_i\right).$$

This again has the usual angular piece of the sensitivity factor. We see, then, that the sensitivity factor is modified. (Unprimed quantities are the "true" ones.)

$$K_n = K_n^{'}\left(1-\frac{n^2}{6}\sum_{i=1}^{3}\varepsilon_i\right).$$

The harmonic amplitude is calculated from the flux using the unmodified sensitivity factor introducing an error in the result. The ratio of the calculated harmonic amplitude *C'(n)* to the true one *C(n)* is given by

$$\frac{C^{'}(n)}{C(n)} = 1 - \frac{n^2}{6}\sum_{i=1}^{3}\varepsilon_i .$$

This ratio for the specific case we are discussing differs from 1 by 0.04% for n=1 to 0.9% for n=10.

| n | C'(n)/C(n) |
|---|---|
| 1 | 0.9999 |
| 2 | 0.9997 |
| 3 | 0.9992 |
| 4 | 0.9986 |
| 5 | 0.9978 |
| 6 | 0.9969 |
| 7 | 0.9958 |
| 8 | 0.9945 |
| 9 | 0.9930 |
| 10 | 0.9914 |

Perhaps more interesting is the error in normalized harmonic which I will calculate here for a quadrupole.

$$c'(n) = \frac{C^{'}(n)}{C^{'}(2)} = \frac{C(n)\left(1 - \frac{n^2}{6}\sum_{i=1}^{3}\varepsilon_i\right)}{C(2)\left(1 - \frac{2}{3}\sum_{i=1}^{3}\varepsilon_i\right)} = c(n)\frac{1 - \frac{n^2}{6}\sum_{i=1}^{3}\varepsilon_i}{1 - \frac{2}{3}\sum_{i=1}^{3}\varepsilon_i}; \frac{c'(n)}{c(n)} = \frac{1 - \frac{n^2}{6}\sum_{i=1}^{3}\varepsilon_i}{1 - \frac{2}{3}\sum_{i=1}^{3}\varepsilon_i}.$$

This increases the error slightly. (The incorrect measurement of the higher order harmonic is normalized to an incorrect value of the main field.)

| n | c'(n)/c(n) |
|---|---|
| 1 | - |
| 2 | - |
| 3 | 0.9996 |
| 4 | 0.9990 |
| 5 | 0.9982 |
| 6 | 0.9972 |
| 7 | 0.9961 |
| 8 | 0.9948 |
| 9 | 0.9933 |
| 10 | 0.9917 |

Note that there is also an effect on the magnitude of the centering offset $z_0$.

$$| z_0 | \propto \frac{C^{'}(1)}{C^{'}(2)}.$$

$$\frac{|z_0^{'}|}{|z_0|}=\frac{1-\frac{1}{6}\sum_{i=1}^{3}\varepsilon_i}{1-\frac{2}{3}\sum_{i=1}^{3}\varepsilon_i}.$$

In this particular case, the error introduced is 0.03%.

## *A discussion of various twist effects*

Let's consider three examples of twist.

The first is a discrete offset between two sections of a probe body. This is the case we discuss above. For discussion we will consider an offset Δ between two probe sections of length L/2.

$$\frac{1}{L}\int_0^L \varepsilon^2(z)dz=\frac{1}{L}\left[\int_0^{L/2}\left(-\frac{\Delta}{2}\right)^2 dz+\int_{L/2}^{L}\left(\frac{\Delta}{2}\right)^2 dz\right]=\frac{\Delta^2}{4}.$$

The ratio of the sensitivity factor to the ideal one is

$$\frac{K_n}{K_n^{ideal}}=1-\frac{n^2}{8}\Delta^2.$$

The second case will be a systematic twist of Δ from one end of the probe form to the other. The probe length is L. We need to evaluate the same integral as before with $\varepsilon(z)$ given by $\Delta/L\cdot z$.

$$\frac{1}{L}\int_0^L \varepsilon^2(z)dz=\frac{1}{L}\int_0^L\left(\frac{\Delta}{L}z\right)^2 dz=\frac{\Delta^2}{3}.$$

The ratio of the sensitivity factor to the ideal one is

$$\frac{K_n}{K_n^{ideal}}=1-\frac{n^2}{6}\Delta^2.$$

The last case is the one discussed in [1]. We have a random twist characterized by a deviation Δ such that

$$\Delta^2=\frac{1}{L}\int_0^L\left(\delta_c-\delta_z\right)^2 dz.$$

In this case the deviation from the ideal sensitivity factor is given by

$$\frac{K_n}{K_n^{ideal}} = 1 - \frac{n^2}{2}\Delta^2.$$

One can see that in some sense the worst case is the random variation. This presumes that the probable errors in the 3 cases are likely to be of the same size, not necessarily the most likely scenario.

## References

[1] A. Jain, “Harmonic coils”, proceedings of the CERN Accelerator School, Capri, Italy, 1998.